

M.Phil./Ph.D. Entrance Examination, August - 2018
MATHEMATICS**Day and Date : Friday, 10 - 08 - 2018****Total Marks : 100****Time : 1.00 p.m. to 3.00 p.m.****Instructions :**

- 1) All questions are compulsory.
- 2) Each question carries 2 marks.
- 3) Answers should be marked in the given OMR answer sheet by darkening the appropriate option.
- 4) Use black ball point pen only for marking the circle. Do not make any stray mark on the OMR Answer Sheet.
- 5) Follow the instructions given on OMR Sheet.
- 6) Rough work shall be done on the sheet provided at the end of question paper.
- 7) Only non programmable calculators are allowed.

- 1) In the process of conducting research ‘Formulation of Hypothesis’ is followed by
 - A) Statement of Objectives
 - B) Analysis of Data
 - C) Selection of Research Tools
 - D) Collection of Data
- 2) Which one of the following is an indication of the quality of a research journal?
 - A) impact factor
 - B) h-index
 - C) g-index
 - D) i10-index
- 3) Which is the main objective of research?
 - A) To review the literature
 - B) To summarize what is already known
 - C) To get an academic degree
 - D) To discover new facts or to make fresh interpretation of known facts

- 4) The research stream of immediate application is
- A) Conceptual research B) Action research
C) Fundamental research D) Empirical research
- 5) Fundamental research reflects the ability to :
- A) Synthesize new ideals
B) Exound new principles
C) Evaluate the existing material concerning research
D) Study the existing literature regarding various topics
- 6) Research ethics include
- A) punctuality B) subjectivity
C) integrity D) smartness
- 7) An example of scientific knowledge is
- A) the authority of the prophet or great man
B) social traditions and customs
C) religious scriptures or notes
D) laboratory and field experiments
- 8) The experimental study is based on the law of
- A) single variable
B) replication
C) occupation
D) interest of the subject

14) The following statement is due to _____.

“God exists since mathematics is consistent and the devil exists since we cannot prove it”

- A) E.Galois
B) Weyl Herman
C) Paul Cohen
D) Einstein

15) Which of the following is an EPW.

- A) This matrix is Toeplitz
B) This is Toeplitz matrix
C) This is a Toeplitz matrix
D) This is the Toeplitz matrix

16) Marking for “2-em space” consists of _____ units.

- A) 12
B) 24
C) 36
D) 48

17) Which of the following is an EPW.

- A) The matrix A is expressed as $A = UTV^T$
B) A matrix A is expressed as $A = UTV^T$
C) The matrix A is expressed as $A = UTV^T$
D) The matrix A, is expressed as $A = UTV^T$

18) The input in LaTeX to yield $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^2}}$ is

- A) $\$\\displaystyle \\int_0^1 \\div\\{x^2\\, dx\\} \\{\\sqrt\\{1-x^2\\}\\}\\$$
B) $\$\\displaystyle \\int_0^1 \\{x^2\\, dx\\} \\div\\{\\sqrt\\{1-x^2\\}\\}\\$$
C) $\$\\displaystyle \\int_0^1 \\frac{x^2\\, dx\\} \\{\\sqrt\\{1-x^2\\}\\}\\$$
D) $\$\\displaystyle \\int_0^1 \\frac{x^2\\, dx\\} \\{\\sqrt\\{1-x^2\\}\\}\\$$

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- 19) The command to inserts one inch of vertical space after the line in which it appears is
- A) $\backslash vspace*{1in}$ B) $\backslash space*{1in}$
C) $\backslash space*{1in}v$ D) $\backslash v{1in}space*$
- 20) The identity $R_{ikl}^j = -R_{ikl}^j$ can be obtained by typing
- A) $\$ R_i_{k1}^j = -R_j_{ik1} \$$
B) $\$ R^j_i_{k1} = -R^j_{ik1} \$$
C) $\$ R_i_{j_{k1}} = -R^j_{ik1} \$$
D) $\$ R_i_{k1}^j = -R_{j_{ik1}} \$$
- 21) The input in LaTeX
 $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$
yields
- A) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ B)

C)
- 22) Scilab is _____.
A) Open Source
B) Commercial but cheaper than Matlab
C) Commercial but cheaper than Maple
D) Commercial but cheaper than mathematica
- 23) Files having the _____ extension contain both Scilab functions and executable statements.
A) B)
C) D)

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24) Imaginary number i is represented in Scilab by using _____.

- | | |
|----------|--------|
| A) I | B) i |
| C) imath | D) %i |

25) What will be the output of following Scilab commands?

$A = \text{ones}(3, 3); A(2, :) =$

- | | |
|----------|----------|
| A) 1 1 1 | B) 3 3 3 |
| C) 1 2 3 | D) 2 2 2 |

26) where L is any rectifiable arc joining the points $z = a$ and $z = b$ is equal to

- | | |
|--------------------|------------|
| A) $b-a$ | B) $ b-a $ |
| C) arc length of L | D) 0 |

27) The radius of convergence of the power series of the function

about $z = 1/4$ is $\oint_L dz/(z-1)^{-1} = (1-z)^{-1}$

- | | |
|--------|--------|
| A) 1 | B) 1/4 |
| C) 3/4 | D) 0 |

28) Let $\mathcal{P} = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}, X\}$ where $X = \{a,b,c,d\}$. Then which one of the following sets is a base for \mathcal{P} .

- | | |
|--|--|
| A) $\beta = \{\{a\}, \{b\}, \{c,d\}\}$ | B) $\beta = \{\{a\}, \{c\}, \{c,d\}\}$ |
| C) $\beta = \{\{c\}, \{b\}, \{c,d\}\}$ | D) $\beta = \{\{a\}, \{b\}\}$ |

29) Which of the following is true? A subset A of R^n is compact if:

- A) the complement of A is an open set
- B) A is connected and closed
- C) A is closed and bounded
- D) every limit point of A is in A

- 30) Which of the following statement is false?
- A) The product of two orthogonal matrices is orthogonal
 - B) Inverse of a non-singular Hermitian matrix is Hermitian
 - C) The product of two Hermitian matrices is Hermitian
 - D) The product of two unitary matrices is unitary
- 31) In which of the following alternatives a subset T of a set $S = \{(2, 0, 0), (2, 2, 2), (2, 2, 0), (0, 2, 0)\}$ is not a basis of $R^3 (R)$?
- A) $T = \{(2, 0, 0), (2, 2, 0), (2, 2, 2)\}$
 - B) $T = \{(2, 0, 0), (2, 2, 2), (0, 2, 0)\}$
 - C) $T = \{(2, 0, 0), (2, 2, 0), (0, 2, 0)\}$
 - D) $T = \{(2, 2, 0), (2, 2, 2), (0, 2, 0)\}$
- 32) Let G be a graph with 5 vertices v_1, v_2, v_3, v_4 and v_5 such that $d(v_1) = 3$, $d(v_2) = 4$, $d(v_3) = 3$, $d(v_4) = 3$, $d(v_5) = 1$ then the number edges in G are ____.
- A) 28
 - B) 14
 - C) 7
 - D) 30
- 33) If $A(x) = 1 + x^3 + x^4 + x^8$ and $B(x) = 1 + x^4 + x^5 + x^8$ then the coefficient of x^8 in $A(x).B(x)$ is _____.
- A) 3
 - B) 4
 - C) 5
 - D) 6

34) If \mathbb{Q} is a field of rational numbers then _____.

- | | |
|------|-------|
| A) 2 | B) 15 |
| C) 4 | D) 6 |

35) A field of characteristic $p \neq 0$ is perfect if

- A) p^{th} root of any element of F is in F
- B) p^{th} power of any element of F is in F
- C)
- D) $a^{p^m} \in F$ for all positive integers m and for all $a \in F$

36) The series _____, converges uniformly on _____.

- | | |
|-------------|-------------|
| A) $[0, 1]$ | B) $[0, 1)$ |
| C) $(0, 1]$ | D) |

37) For a vector field $\mathbf{F}(x, y, z) = \langle x^2, y^2, z^2 \rangle$, the trace of the Jacobian matrix of \mathbf{F} is _____.

- A) curl
- B) grad
- C) div
- D)

38) If G is a non abelian group of order p^3 where p is prime then $O(Z(G)) =$ _____.

- | | |
|----------|----------|
| A) 3 | B) p |
| C) p^3 | D) p^2 |

39) Let M be an R -module. Consider the following two statements.

- I) Any intersection of submodules of M is a sub-module of M .
- II) Finite union of submodules of M is a sub-module of M .

Then

- A) Only (I) is true
- B) Only (II) is true
- C) Both (I) and (II) are true
- D) Both (I) and (II) are false

40) All the eigen values of the matrix lie in the interval

- | | |
|------------|-----------|
| A) (-4, 6) | B) (0, 8) |
| C) (-4, 8) | D) (0, 6) |

41) By trapezoidal rule

- | | |
|------------|-------------|
| A) 0.75 | B) .6944 |
| C) $\ln 2$ | D) $\log 2$ |

42) For equation $x^3(x-1)y'' + (x-1)x^2y' + y = 0, 2y = 3$.

$$\begin{array}{c} \boxed{x^3(x-1)} \\ \boxed{y''} \\ \boxed{+} \\ \boxed{(x-1)} \\ \boxed{x^2} \\ \boxed{y'} \\ \boxed{+} \\ \boxed{y} \\ \boxed{=} \\ \boxed{0} \\ \boxed{2y} \\ \boxed{=3} \end{array}$$

 A) $x=0$ is regular singular, $x=1$ is irregular singular point.
 B) $x=0, 1$ are irregular singular points.
 C) $x=0, 1$ are regular singular points.
 D) $x=0$ is irregular singular, $x=1$ is regular singular point.

43) Indicial polynomial for the equation is

- | | |
|-------------------|------------------|
| A) $r^2 + r - 1$ | B) $r^2 - 1$ |
| C) $r^2 + 2r - 1$ | D) $r^2 - r + 1$ |

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- 44) If the constraints involved in the system are rheonomic then the kinetic energy is _____ function of generalized velocities.
- A) quadratic but not homogeneous
 - B) quadratic and homogeneous
 - C) linear and homogeneous
 - D) cubic but not homogeneous
- 45) The system for which all the forces (except the forces of constraints) are derivable from a generalized scalar potential U is called _____.
- A) conservative
 - B) monogenic
 - C) monotonic
 - D) scleronomous
- 46) Complete integral of PDE $p + q = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)^2 + \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial z} \right)^2 + 1$
- A)
 - B)
 - C)
 - D)
- 47) Solution of the PDE involving arbitrary constants a, b and c is
- A)
 - B)
 - C)
 - D) $u = a^2x + b^2y + c^2z + abc$

- 48) Let $E = (1, 4]$, $A = (1, 2]$, $B = (2, 3)$, $C = [3, 4]$. Define $\phi = 2\chi_A + 4\chi_B + 5\chi_C$.

Then $\int_E \phi =$

- | | |
|-------|-------|
| A) 87 | B) 44 |
| C) 33 | D) 11 |

- 49) Let $E = (0, 1]$, and for each n , ($n = 1, 2, 3, \dots$) define

Then $\lim_{n \rightarrow \infty} \int_E f_n =$

- | | |
|------|----|
| A) 0 | B) |
| C) | D) |

$$f_n(x) = \begin{cases} 0 & \text{if } x \in \left[0, \frac{1}{n}\right], \\ n & \text{if } x \in \left(\frac{1}{n}, 1\right), \\ \text{does not exist} & \text{if } x \in \left[\frac{1}{n}, 1\right]. \end{cases}$$

- 50) A regular curve α is a reparametrization of a straight line if and only if ____.

- | | |
|------------------|----|
| A) | B) |
| C) is tangent to | D) |



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Rough Work